IVAN FRANKO NATIONAL UNIVERSITY OF LVIV

WORKSHOP ON CURRENT PROBLEMS IN PHYSICS

PROGRAM AND ABSTRACTS

Lviv, 08–09 July 2014

08 July 2014

09:00–09:10 Opening (12, Drahomanov St., auditorium 10)

Chairman: **V. Tkachuk**

09:10–09:50 *O. Derzhko*, On the ferromagnetism of itinerant electrons

09:50–10:30 *A. Rovenchak*, Two-parametric fractional statistics as an effective model of physical systems

10:30–11:10 *B. V. Padlyak, I. I. Kindrat, V. O. Protsiuk, A. Drzewiecki*, Spectroscopy and local structure of the rare-earth impurity ions in lithium tetraborate glasses

11:10–11:40 Coffee-break

Chairman: **A. Rovenchak**

11:40–12:20 *B. Novosyadlyj, O. Sergijenko*, Dynamical dark energy after Planck-2013 results: advances and problems

12:20–13:00 *A. Duviryak*, Almost-circular orbit method for quantization of the Fokker action integrals

Lunch

Chairman: **A. Duviryak**

14:40–15:00 *Yu. A. Mishchenko, A. M. Gavrilik*, Thermodynamics aspects and correlation function intercepts in $\tilde{\mu}, q$-deformed Bose gas models

15:00–15:20 *A. Kuzmak*, Geometry of the manifold of eigenstates of the operator of projection of spin-s on an arbitrary direction

15:20–15:40 *I. R. Dulepa, O. V. Velychko*, Investigation of boson spectrum of two-dimensional optical lattices with the graphene type structure

15:40–16:00 Coffee-break

16:00–16:20 *Yu. Yaremko*, Relativistic dynamics of a charge in a Penning trap

16:20–16:40 *I. I. Kindrat, B. V. Padlyak, A. Drzewiecki*, The EPR and optical spectroscopy of the Sm-doped borate glasses

16:40–17:00 *A. V. Nazarenko*, Area quantization of the parameter space of the Riemann surfaces in genus two

09 July 2014

Chairman: **O. Derzhko**

09:00–09:40 *A. Trokhymchuk*, Modern view of liquids: van der Waals and beyond

09:40–10:20 *V. Tkachuk*, Planck-scale effects on quantum and classical systems

10:20–11:00 *M. Stetsko*, Tunneling method and quasi-normal modes for Kaluza–Klein black holes

11:00–11:30 Coffee-break

Chairman: **B. Novosyadlyj**

11:30–12:10 *I. O. Vakarchuk, Iu. M. Diakiv*, Differential cross-section of the photoelectric effect in the deformed electromagnetic field

12:10–12:30 *K. Haydukivska, V. Blavatska*, Universal properties of ring polymers in long-rang correlated disorder

12:30–12:50 *O. Voznyak*, Quasi-exactly solvable potentials with two arbitrary known eigenstates for systems with a position-dependent mass

12:50–13:10 *Kh. P. Gnatenko*, Rotational symmetry in noncommutative space and hydrogen atom

13:10–13:30 *M. Samar*, A dynamical model for the origin of Lorentz-covariant noncommutative spacetime

13:30–. . . Closing

ON THE FERROMAGNETISM OF ITINERANT ELECTRONS

O. Derzhko
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I shall consider the history of understanding of the ferromagnetism of itinerant electrons, review some basic results on the flat-band Mielke–Tasaki scenario for appearance of the ferromagnetic ground states, and discuss how a flat-band Hubbard system, which is not ferromagnetic in the ground state, may become ferromagnetic due to a small dispersion of the flat band in the presence of strong on-site Hubbard correlations [1].

[1] O. Derzhko, J. Richter, Dispersion-driven ferromagnetism in a flat-band Hubbard system, arXiv:1404.2230.

TWO-PARAMETRIC FRACTIONAL STATISTICS AS AN EFFECTIVE MODEL OF PHYSICAL SYSTEMS

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A two-fold parametrization of the expression for the distribution function (occupation numbers) is suggested aiming to find an effective mathematically simple tool for studies of several physical systems. The following two systems are analyzed in detail: weakly-interacting bosons and free anyons. The first parameter of the model is introduced by considering the Polychronakos and Haldane–Wu statistics. The second one is obtained by modifying the ordinary exponential in the Boltzmann factor. It can be made within the nonextensive statistics (using the Tsallis q-exponential) or within the so called incomplete statistics.

In the case of the weakly-interacting Bose-system, the parameters of the introduced fractional statistics models are linked to effects of interactions as well as finite-size corrections. The behavior of the specific heat and condensate fraction of three-dimensional isotropic harmonic oscillators with respect to the values of the statistics parameters is studied in the temperature domain including the BEC-like phase transition point [1].

An approximate correspondence is shown to hold between the anyonic statistics and three two-parametric fractional statistics models, namely the nonextensive Polychronakos statistics and both the incomplete and the nonextensive modifications of the Haldane–Wu statistics. The values of the parameters are linked to the anyonic parameter α using the expressions of the second and third virial coefficients. It is suggested that the search for the expression of the anyonic distribution function should be made within some modifications of the Haldane–Wu statistics [2].

[1] A. Rovenchak, Phys. Rev. A **89**, 052116 (2014).
[2] A. Rovenchak, preprint arXiv:1403.3577, to appear in Eur. Phys. J. B (2014).

SPECTROSCOPY AND LOCAL STRUCTURE OF THE RARE-EARTH IMPURITY IONS IN LITHIUM TETRABORATE GLASSES

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The electron paramagnetic resonance (EPR), ground state optical absorption and photoluminescence (emission and excitation) spectra as well as luminescence kinetics of the Nd-, Er-, Dy-, and Gd-doped lithium tetraborate ($Li_2B_4O_7$) glass have been investigated and analysed. The rare-earth doped lithium tetraborate glasses were obtained from corresponding polycrystalline compounds using standard glass technology [1-4]. The EPR and optical spectroscopy shows that the rare-earth impurities are incorporated into the $Li_2B_4O_7$ glass network of as trivalent ions (Nd^{3+}, Er^{3+}, Dy^{3+}, and Gd^{3+}), exclusively, which reveal in their characteristic EPR and optical (absorption, emission, and luminescence excitation) spectra as well as luminescence kinetics. In the EPR spectra of all investigated glasses have been observed characteristic signals, which were assigned to the $Nd3^{3+}$, Er^{3+}, Dy^{3+}, and Gd^{3+} isolated centres and corresponding rare-earth pair centres, coupled by magnetic dipolar and exchange interactions. All optical bands of the Nd^{3+}, Er^{3+}, and Dy^{3+} rare-earth centres in the optical absorption, emission and luminescence excitation spectra in the $Li_2B_4O_7$ glasses were identified. The luminescence decay curves for $Nd3^{3+}$, Er^{3+}, and Dy^{3+} centres in the $Li_2B_4O_7$ glasses were fitted by exponential function with corresponding lifetimes, which were determined at $T = 300$ K.

Based on the EPR and optical spectroscopy results [2–4] supported by XRD structural data for $Li_2B_4O_7$ glass and crystal [1] the local structure of rare-earth impurity centres in the Li^+ cationic sites was proposed. The proposed local structure for Nd^{3+}, Er^{3+}, Dy^{3+} and Gd^{3+} ions is confirmed in [5] by direct Extended X-ray Absorption Fine Structure (EXAFS) investigation and analysis of the L_3 edge of Nd, Er, Dy, and Gd impurities in the $Li_2B_4O_7$ glass network.

[1] B. V. Padlyak, S. I. Mudry, Y. O. Kulyk, A. Drzewiecki, V. T. Adamiv, Y. V. Burak, I. M. Teslyuk, Mater. Sci. Poland **30**, 264 (2012).

[2] B. V. Padlyak, W. Ryba-Romanowski, R. Lisiecki, V. T. Adamiv, Ya. V. Burak, I. M. Teslyuk, Proc. of the Int. Conf. on Oxide Materials for Electron. Engneering (OMEE-2012) (National University "Lviv Polytechnic", Lviv, 2012), p. 200–201.

[3] B. Padlyak, A. Drzewiecki, J. Non-Cryst. Solids **367**, 58 (2013).

[4] B. V. Padlyak, I. I. Kindrat, V. O. Protsiuk, A. Drzewiecki, Ukr. J. Phys. Opt. **15**, 103 (2014).

[5] T. D. Kelly *et al.*, Front. Phys.: Cond. Matt. Phys. **2**, (Art. 31) 1 (2014).

DYNAMICAL DARK ENERGY AFTER PLANCK-2013 RESULTS: ADVANCES AND PROBLEMS

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We analyze the possibility of reconstruction of scalar field dark energy and determination of its parameters: density $\Omega_{\rm de}$, equation of state parameter w_0 and effective sound speed c_s together with other cosmological parameters on basis of the dataset including Planck-2013 results on CMB anisotropy, BAO distance ratios from recent galaxy surveys, magnitude-redshift relations for distant SNe Ia from SNLS3 sample and the HST determination of the Hubble constant. Using the Markov Chain Monte Carlo routine to map out the likelihood in the multi-dimensional parameter space we have obtained the best-fit values and 2σ confidence limits for the main dark energy and cosmological parameters. It is shown that phantom scalar field model of dark energy is strongly preferred by this dataset ($-1.2 < w_0 < -1.04$), so, the ΛCDM model is disfavored at 2σ confidence level. On the other hand, to reconstruct the Lagrangian we need to know the value of effective sound speed which is determined badly: no value of c_s from the range [0,1] is preferred by this dataset because of the very weak influence of dark energy perturbations on the large scale structure formation and CMB temperature fluctuations. If the phantom type of dark energy will be confirmed then for reliable reconstruction of dark energy not only high accuracy data but also radically new ideas for its study would be necessary.

ALMOST-CIRCULAR ORBIT METHOD FOR QUANTIZATION OF THE FOKKER ACTION INTEGRALS

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Fokker-type action integrals [1, 2] represent one of the action-at-a-distance approaches to the relativistic dynamics of interacting particles. They include semiphenomenological models and provide an alternative (or complementary) description of known field-theoretical interactions, such as well the Wheeler–Feynman electrodynamics [3], the gravitation [3, 4], confining interactions [5, 6] etc [7, 8].

A variational problem based on the Fokker-type action describes a dynamical system with time non-locality, *i.e.*, it leads to difference-differential or integral-differential equations of motion for which the Cauchy problem is unsuitable. Consequently, the study the phase space (*i.e.*, a set of possible states), the construction of the Hamiltonian description and a quantization of such a system are non-trivial tasks.

Serious effort was made to develop Hamiltonization and quantization procedures for the Fokker-type action integrals. In general, this is attained by means of an approximated reformulation of the problem into another but time-local form [9, 8].

In the present report an alternative approach to this problem is considered. It is applicable to the action principle of Fokker type determining a two-particle time-nonlocal dynamics.

The system is assumed general but invariant with respect to the Aristotle group which is a common subgroup of the Galileo and Poincaré groups. By this both non-relativistic and relativistic cases are included.

It is shown that integral-differential equations of motion of the system admit circular orbit solutions. For this purpose the uniformly rotating reference frame is used. Then the dynamics of perturbations of circular solutions is derived and studied. On this ground the Hamiltonian description of the time-nonlocal two-particle system is built in the almost circular orbit (ACO) approximation. The Aristotle-invariance of the system is exploit in order to select physical degrees of freedom. They are subjected to a canonical quantization procedure. Then a construction of the energy spectrum of the system is proposed.

The method is applied in meson spectroscopy. Namely, a relativistic Fokker-type quark model of mesons is proposed, in which an interquark interaction is mediated by scalar-vector superposition of higher derivative fields. In the non-relativistic limit the model describes a two-particle system with the linear potential. In order to analyze the model in the essentially relativistic domain the ACO quantization is applied. It is shown that the model reproduces well specific features of the light meson spectroscopy.

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[3] Yu. S. Vladimirov, A. Yu. Turygin, *Theory of direct interparticle interaction* (Energoatomizdat, Moscow, 1986) [in Russian].

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THERMODYNAMICS ASPECTS AND CORRELATION FUNCTION INTERCEPTS IN $\tilde{\mu}, Q$-DEFORMED BOSE GAS MODELS

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Being a deformation (nonlinear extension) of the standard Bose gas model, deformed Bose gas models can effectively take into account different sides of nonideality like in real gases. *Deformed oscillators* are often used as a building block when constructing deformed Bose gas models. Note that deformed oscillators (deformed bosons) can realize [1] composite bosons, in algebraic sense. Some deformed Bose gas models were

applied [2] for thermodynamic description of such aspect of real gases as interaction of particles. In those models, certain thermodynamic or statistical relations of ideal gas undergo deformation (say, by means of Jackson derivative), and the others can be deduced as consequences.

In this work we deal with the $\tilde{\mu}, q$-deformation which combines the quadratically nonlinear $\tilde{\mu}$-deformation appearing in [1], and the Arik-Coon type q-deformation, applied to incorporate [2] interparticle interaction. This combined $\tilde{\mu}, q$-deformation is described by the deformation structure function $\varphi_{\tilde{\mu},q}(n) = (1+\tilde{\mu})[n]_q - \tilde{\mu}([n]_q)^2$ where $[n]_q \equiv (1-q^n)/(1-q)$. The $\tilde{\mu}, q$-deformed Bose gas model, based on the deformed relation for the total number of particles, namely $N_{(\varphi)} = \varphi(z\frac{d}{dz}) \ln Z$ (here the φ-deformed analog $\varphi(z\frac{d}{dz})$ of derivative is used) was considered recently [3]. We give some arguments that the respective deformed virial expansion $\frac{Pv}{k_{\mathrm{B}}T} = \sum_{k=1}^{\infty} V_k(\tilde{\mu}, q)(\frac{\lambda^3}{v})^{k-1}$ can effectively account for certain interparticle interaction jointly with the composite structure of particles of a gas. Besides, for slightly different $\tilde{\mu}, q$-deformed Bose gas model, we calculate the deformed analogs of one- and two-particle distributions $\langle (a_{\mathbf{k}}^{\dagger})^r (a_{\mathbf{k}})^r \rangle$, $r = 1, 2$, as well as 2-particle correlation function intercept $\lambda^{(2)}(\mathbf{k}) = \frac{\langle (a_{\mathbf{k}}^{\dagger})^2 (a_{\mathbf{k}})^2 \rangle}{\langle a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} \rangle^2} - 1$ defined through the creation/annihilation operators of $\tilde{\mu}, q$-boson. The obtained momentum dependencies for the intercepts quantitatively agree with experimental data for π-mesonic correlation intercepts extracted in relativistic heavy ion collisions.

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GEOMETRY OF THE MANIFOLD OF EIGENSTATES OF THE OPERATOR OF PROJECTION OF SPIN-s ON AN ARBITRARY DIRECTION

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We consider the rotation of the eigenstates of the operators of projection of spin-1 and spin-3/2 on the direction $\mathbf{n}$ about the axis directed along some unit vector. It is shown that rotation of such eigenstates takes place on some manifolds defined by two real parameters. Also, we show that the Fubini–Study metrics of these manifolds are the spheres with radii dependent on the value of the spin and on the value of the spin projection. We consider quantum evolution on these manifolds. Finally, we generalize these results for arbitrary spin s.

INVESTIGATION OF BOSON SPECTRUM OF TWO-DIMENSIONAL OPTICAL LATTICES WITH THE GRAPHENE TYPE STRUCTURE

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Energy spectrum of normal and superfluid phases of the Bose-atom system in the optical lattices of the graphene type is investigated. Calculation of dispersion laws in energy bands and single-particle spectral densities is performed in the random phase approximation on the basis of the hard-core boson formalism. Their changes at the transition from normal phase to superfluid one are described. During such a transformation the number of subbands is doubled; in the case of energetic equivalence of subbands the Dirac points in the spectrum are conserved but their number increases twice. The temperature-dependent gapless spectrum with Dirac points placed on the border of Brillouin zone is obtained for the lattice with energetically equivalent sites. Chemical potential is located outside of the allowed energy band. A difference between site energies of particles in different sublattices causes a gap in the spectrum and chemical potential can be located between subbands. It is shown that the shape of spectral densities is sensitive to the change of temperature and the site energy (Figure 1).

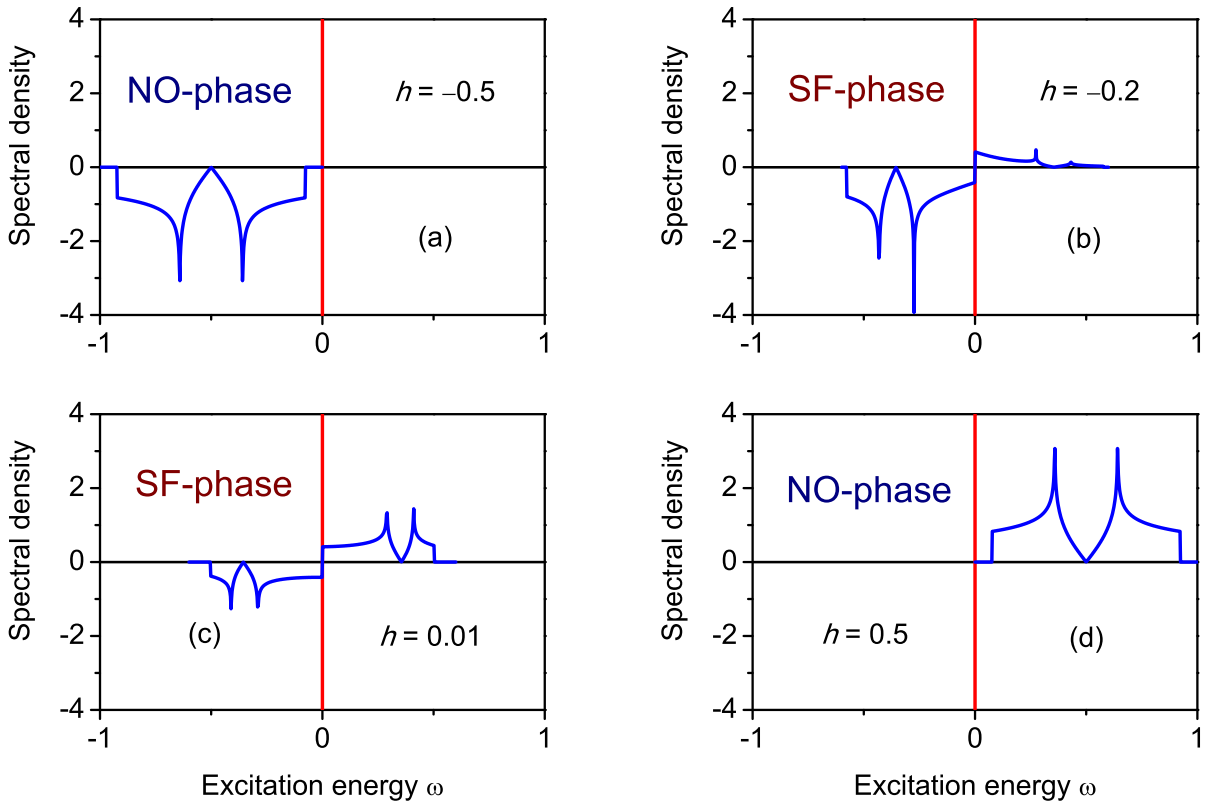

Figure 1: Spectral density in NO- and SF-phases for energetically equivalent sublattices as a function of excitation energy for a sequence of values of the site energy h ((a)–(d)) at chemical potential $\mu = 0$ and temperature $\Theta = 0.2$ (all energetic parameter are given in units of transfer energy $J(0) = 1$).

RELATIVISTIC DYNAMICS OF A CHARGE IN A PENNING TRAP

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We are interested in the motion of a classical charge within a processing chamber of a Penning trap [1]. We examine the relativistic Lagrangian and Hamiltonian dynamics and show that the radial and axial motions are coupled to each other whenever the special relativity is taken into account. Because of the axial symmetry of restoring quadruple potential we reduce two redundant degrees of freedom and construct the four-dimensional Hamiltonian dynamical system. In the specific case of zero axial oscillation the equations of motion are solved. The solution is expressed in terms of Jacobi elliptic functions. If all the energy of a charge belongs to the axial oscillating mode, its time evolution is described by the nonlinearized equation of motion for a simple pendulum. If the energy flows alternatively between axial and radial oscillating modes, the dynamical system resembles a double pendulum [2].

[1] S. Brown, G. Gabrielse, Rev. Mod. Phys. **58**, 233 (1986)
[2] G. L. Baker, J. A. Blackburn, *The Pendulum. A Case Study in Physics* (Oxford Univ. Press, 2005).

THE EPR AND OPTICAL SPECTROSCOPY OF THE Sm-DOPED BORATE GLASSES

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The electron paramagnetic resonance (EPR) and optical (absorption, photoluminescence emission and excitation) spectra as well as luminescence kinetics of the $Li_2B_4O_7$:Sm, $LiKB_4O_7$:Sm, CaB_4O_7:Sm and $LiCaBO_3$:Sm are investigated and analysed. The Sm-doped borate glasses of high chemical purity and optical quality have been obtained from corresponding polycrystalline compounds in the air atmosphere, using standard glass synthesis according to [1]. The Sm impurity was introduced into the borate polycrystalline compounds in the form of Sm_2O_3 oxide in amounts 0.5 and 1.0 mol. %.

Based on the EPR and optical spectra analysis it was shown that the Sm impurity is incorporated into the borate glass network as Sm^{3+} ($4f^5$, $^6H_{5/2}$) ions, exclusively. In EPR spectra of the Sm-doped borate glasses besides Fe^{3+} non-controlled impurity signals at low temperatures (4.2 ÷ 20 K) have been clearly observed two additional signals, which are assigned to the Sm^{3+} isolated centres and Sm^{3+}–Sm^{3+} pair centres, coupled by magnetic dipolar and exchange interactions. All $4f$–$4f$ transitions of the Sm^{3+} ions, observed in the optical absorption and photoluminescence spectra have been

identified. Typical optical absorption spectrum of the Sm-doped borate glasses consists of intense broad band (fundamental absorption edge of the glass host), several weak bands in the visible spectral range, and several intense bands in the infrared spectral range. The photoluminescence spectra of the $Li_2B_4O_7$:Sm, $LiKB_4O_7$:Sm, CaB_4O_7:Sm and $LiCaBO_3$:Sm glasses containing 0.5 and 1.0 mol.% Sm_2O_3 registered under excitation with λ_{exc}=401 nm ($^6H_{5/2} \rightarrow {}^6P_{3/2}$ absorption transition) at T =300 K are closely similar and contain 3 characteristic emission bands peaked about 562, 598 and 645 nm, which correspond to the $^4G_{5/2} \rightarrow {}^6H_{5/2}$, $^6H_{7/2}$, $^6H_{9/2}$ transitions of the Sm^{3+} centres, respectively. Luminescence excitation spectra of the Sm-doped borate glasses consist of number weakly-resolved bands, which show good correlation with corresponding optical absorption bands. Weak resolution of some bands is related to inhomogeneous broadening caused by structural disordering of the glass host.

Luminescence kinetics for Sm^{3+} centres in the borate glasses are satisfactorily described by single exponential decay with lifetimes 2.65 and 2.57 ms ($Li_2B_4O_7$:Sm glass), 2.78 and 2.68 ms ($LiKB_4O_7$:Sm glass), 2.52 and 2.37 ms (CaB_4O_7:Sm glass), and 2.31 and 2.13 ms ($LiCaBO_3$:Sm glass) for samples, containing 0.5 and 1.0 mol. % Sm_2O_3, respectively. Various lifetimes for Sm^{3+} centres in different borate glasses are caused by some differences in the local structure of Sm^{3+} luminescence centres in the network of investigated borate glasses. The resonance energy transfer through cross-relaxation processes between the Sm^{3+}–Sm^{3+} pair centres coupled by electric multipolar interactions is responsible for quenching of lifetimes in the glass samples contained relatively high (1 mol. %) concentration of the Sm impurity.

The peculiarities of electronic and local structure of the Sm^{3+} centres in the $Li_2B_4O_7$:Sm, $LiKB_4O_7$:Sm, CaB_4O_7:Sm and $LiCaBO_3$:Sm glasses have been discussed in comparison with referenced data for Sm-doped borate glasses and crystals with similar chemical compositions as well as with other borate glasses, doped with Sm.

[1] B. V. Padlyak, S. I. Mudry, Y. O. Kulyk, A. Drzewiecki, V. T. Adamiv, Y. V. Burak, I. M. Teslyuk, Mater. Sci. Poland **30**, 264 (2012).

AREA QUANTIZATION OF THE PARAMETER SPACE OF THE RIEMANN SURFACES IN GENUS TWO

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It is explored a model of compact Riemann surfaces in genus two, represented geometrically by two-parametric hyperbolic octagons with an order four automorphism. We compute the generators of associated isometry group (Fuchsian group) and give a real-analytic description of corresponding Teichmüller space, parametrized by the Fenchel-Nielsen variables (lengths and twists arising in pant decomposition of the surface), in terms of geometric data. Using the Weil–Petersson geometry and Wolpert's

formula, we compute symplectic two-form in parameter space. Moreover, it turns out that this symplectic manifold can be densely covered by the orbits of constant perimeter of octagons (isoperimetric orbits). This fact permits us to introduce the action and angle variables, where the former are related to the Weil-Petersson area in parameter space. In these terms there is a possibility to evaluate the number of quantum "cells" inside domain bounded by isoperimetric orbits. Further, we apply our formalism to quantize a physical system, namely, the model with SU(1,1) symmetry, corresponding to (2+1)-dimensional loop gravity, inspiring us by the known results of area quantization.

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MODERN VIEW OF LIQUIDS: VAN DER WAALS AND BEYOND

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It is generally assumed that the regular van der Waals theory is useful above the critical point temperatures, where it serves as an improvement of the ideal gas law. The van der Waals theory still can be applied for subcritical temperatures as well, where it continues to be qualitatively reasonable for the liquid states and the low-pressure gaseous states. However, the regular van der Waals theory is not appropriate for the rigorous quantitative calculations, thus remaining to be useful for teaching and qualitative purposes only.

Here we are arguing, that there exists way to separate the total pair interaction into two parts, namely, into the short-range interaction (includes both repulsion and attraction) and the long-range interaction, that do allow for the van der Waals theory to be used for accurate quantitative calculations as well. These our ideas are illustrated by the calculations performed for the Sutherland and Lennard-Jones potentials. The applications of this approach to other kinds of fluids, like ionic and associating fluids, including water, are discussed as well.

PLANCK-SCALE EFFECTS ON QUANTUM AND CLASSICAL SYSTEMS

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The investigations in string theory and quantum gravity suggest the existence of the nonzero minimal uncertainty in position (minimal length) which is of the order of the Planck scale. The minimal length can be described in the frame of deformed commutation relation between operator of position and momentum. Note that the deformation of the Heisenberg algebra and the deformation of the Poisson brackets in the classical case bring not only technical difficulties in solving of corresponding equations but also bring problems of a fundamental nature. The equivalence principle, gauge invariance, Galilean and Lorentz transformations, gauge invariance in the space with minimal length will be discussed in this talk.

TUNNELING METHOD AND QUASI-NORMAL MODES FOR KALUZA–KLEIN BLACK HOLES

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Ivan Franko National University of Lviv, Ukraine

Tunneling method was applied to Kaluza–Klein black holes. Temperature of the black holes is obtained for scalar particles as well as for fermions. We obtained identical expressions for temperature for both cases and this result coincides with results obtained earlier by methods of QFT in curved space-time. Quasi-normal modes methods was also considered. Quasi-normal frequencies are calculated and analyzed.

DIFFERENTIAL CROSS-SECTION OF THE PHOTOELECTRIC EFFECT IN THE DEFORMED ELECTROMAGNETIC FIELD

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We calculate the differential cross-section of photoelectric effect of system of non-linear quantum field, where non-linearity is due to deformation of the Poison brackets of the generalized coordinates and momenta. For $\beta \to \infty$ differential cross-section reduces to the well-known expression of differential cross-section of photoelectronic effect for non-deformed field. In the case of considerable deformation ($\beta \to \infty$)differential cross-section of photoelectronic effect decreases as β^{-3}.

UNIVERSAL PROPERTIES OF RING POLYMERS IN LONG-RANG CORRELATED DISORDER

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We analyze the universal size and shape characteristics of flexible ring polymers in solutions in presence of structural obstacles (impurities) in d dimensions. A special case when impurities are correlated on large distances r according to a power law $\sim r^{-a}$ is considered. Applying the direct polymer renormalization scheme, we evaluate the estimates for averaged gyration radius $\langle R_{g\,\mathrm{ring}}\rangle$ and spanning radius $\langle R_{1/2\,\mathrm{ring}}\rangle$ of typical ring polymer conformation up to the first order of double $\varepsilon = 4-d$, $\delta = 4-a$ expansion. Our results quantitatively reveal an extent of an effective size and anisotropy of closed ring macromolecules in disordered environment. In particular, the size ratio of ring and open (linear) polymers of the same molecular weight grows when increasing the strength of disorder according to $\langle R^2_{g\,\mathrm{ring}}\rangle/\langle R^2_{g\,\mathrm{chain}}\rangle = \frac{1}{2}\left(1+\frac{13}{48}\delta\right)$.

QUASI-EXACTLY SOLVABLE POTENTIALS WITH TWO ARBITRARY KNOWN EIGENSTATES FOR SYSTEMS WITH A POSITION-DEPENDENT MASS

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The method of supersymmetric quantum mechanics has been applied to construct quasi-exactly solvable potentials for systems with a position-dependent mass. The conditions which provide regular potential energy in the case of singular generating function have been established. The examples of the quasi-exactly solvable potentials with two known eigenstates have been considered in the cases of regular and singular generating functions and the corresponding eigenfunctions have been found exactly.

ROTATIONAL SYMMETRY IN NONCOMMUTATIVE SPACE AND HYDROGEN ATOM

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We consider the problem of rotational symmetry breaking in three-dimensional noncommutative space

$$[X_i, X_j] = i\hbar\theta_{ij}, \tag{1}$$

where θ_{ij} is a constant antisymmetric object. In order to solve this problem the generalization of the parameter of noncommutativity is proposed

$$\theta_{ij} = \frac{\alpha}{\hbar}(a_i b_j - a_j b_i), \tag{2}$$

where α is a dimensionless constant, and a_i, b_i are governed by the harmonic oscillators

$$H_{osc} = \frac{(p^a)^2}{2m} + \frac{(p^b)^2}{2m} + \frac{m\omega^2 a^2}{2} + \frac{m\omega^2 b^2}{2}. \tag{3}$$

Therefore, we construct the noncommutative algebra which is rotationally invariant

$$[X_i, X_j] = i\alpha(a_i b_j - a_j b_i). \tag{4}$$

The hydrogen atom is considered in rotationally invariant noncommutative space. We find the corrections to the energy levels of this atom up to the second order in parameter of noncommutativity.

A DYNAMICAL MODEL FOR THE ORIGIN OF LORENTZ-COVARIANT NONCOMMUTATIVE SPACETIME

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We study generalized Lorentz-covariant deformed algebra. This algebra contains those of Snyder [1] and Quesne–Tkachuk [2] as partial cases. We demand the action of a relativistic particle to be invariant under the deformed Poincaré symmetry corresponding to the chosen algebra. It is shown that the Dirac constraint analysis of the model yields the clasical version of the algebra. In the case of the considered algebra leading to Snyder's one, our results coincide with those obtained in [3].

[1] H. S. Snyder, Phys. Rev. **71**, 38 (1947)
[2] C. Quesne, V. M. Tkachuk, J. Phys. A **39**, 10909 (2006).
[3] R. Banerjee *et al.*, JHEP 05, 077 (2006).